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SIMPLE RULE OF SELECTION OF SUBSAMPLING RATIOS FOR WARPED FILTER BANKS*)

This paper considers a subsampling in warped digital filter banks. In such systems non-uniform subband decompositions are easily obtainable (e.g. approximating critical bands of hearing). In the majority of practical applications, signals in each filter bank channel should be subsampled for efficiency reasons. However, contrary to uniform systems and their non-uniform descendants derived by tree-structuring, where critical sampling is possible, additional constraints arise here. Namely, the location of the subband with respect to integer positioning determined by the assumed subsampling ratio is crucial for exact reconstruction. In this contribution the above problem is approached in the context of the bandpass sampling theorem which gives an explicit rule of the careful selection of subsampling ratios for given subband distribution.

1 Essentials of bandpass sampling

When sampling bandpass signal spanning the bandwidth B between f_L and f_U, its frequency location can be given as the band position [1] - the fractional number of bandwidths from the origin at which the lower band edge resides.

There are two special cases of band positioning

$$f_L = c(f_U - f_L) \quad \text{- integer} \qquad (1a)$$

$$f_L = \left(\frac{2c+1}{2}\right)(f_U - f_L) \quad \text{- half-integer} \qquad (1b)$$

where c is a nonnegative integer.

The classical bandpass sampling theorem [1] determines the minimal uniform sampling rate

$$f_s \geq 2\frac{f_U}{n} \qquad n = \left\lfloor \frac{f_U}{B} \right\rfloor \qquad (2)$$

guarantying correct reconstruction of the processed signal.

The absolutely minimal rate

$$f_s^{MIN} = 2B \qquad (3)$$

applies only to integer band positioning. Moreover for bandpass signal, contrary to the lowpass case, not all frequencies satisfying (2) are valid due to the aliasing caused by sampling process.

Namely, all acceptable uniform sampling rates must satisfy the inequalities

$$2\frac{f_U}{n} \leq f_s \leq 2\frac{f_L}{n-1} \qquad 1 \leq n \leq \left\lfloor \frac{f_U}{B} \right\rfloor \qquad (4)$$

ensuring that no irreversible spectral overlapping occurs. They can be clearly visualized in 2D [1] as Fig. 1 shows.

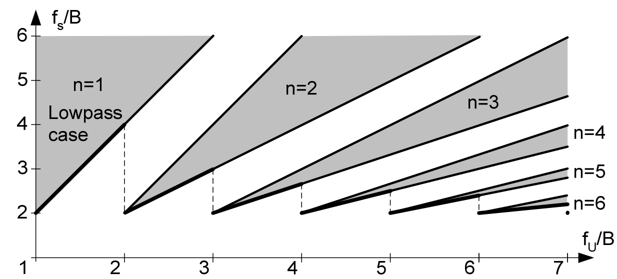

Fig. 1 Allowed sampling rates vs. band position

The shaded wedges are the allowed zones for sampling without aliasing. The remaining area represents disallowed rates resulting in distortions.

The minimal sampling rates (2) lie on the bottom edges of the wedges, on the bold sections cut by the dashed vertical lines. In turn f_s^{MIN} occurs at the tips of the wedges. Given bandpass signal the corresponding vertical line can be drawn revealing allowed sampling rates when intersecting the wedges.

One should mention that there are less conventional non-uniform sampling schemes well suited to (multi-)band signals [2]. However, this topic remains beyond the scope of our considerations.

2 Warped versus uniform filter banks

Most of the contemporary filter banks [3] are multirate systems exploiting polyphase decomposition and modulation via fast transforms. The entire set of filters is derived from only one prototype. However, regardless of the particular structure, the bulk of these systems can be reduced to a set of ordinary bandpass filters with accompanying decimators and expanders as depicted in Fig. 2.

The most common are various variants of uniform DFT and cosine modulated filter banks. In these systems, critical sampling with the rate $S_0 = ... = S_{K-1} = K$ can be performed with negligible (or even without) distortions, depending on the design of the prototype filter. This is owing to the fact that all bands have equal widths, they are half- or integer positioned, and the channel signals are complex- or real-valued accordingly.

These systems can be tree-structured to give non-uniform spectral decompositions with good properties but somewhat limited flexibility, because the bounds of the resulting subbands are constrained

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to that of the (half-)integer positions of underlying uniform subbands.

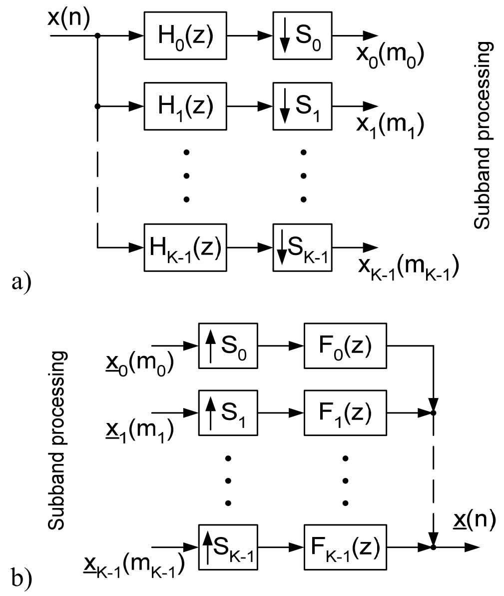


Fig. 2 General schema of analysis (a) and synthesis (b) multirate filter bank

A more flexible alternative is to use warped filter banks [4] to do nonuniform subband decomposition. In these systems the deformation of the frequency axis is exploited to change simultaneously the locations and widths of subbands as shown in Fig. 3.

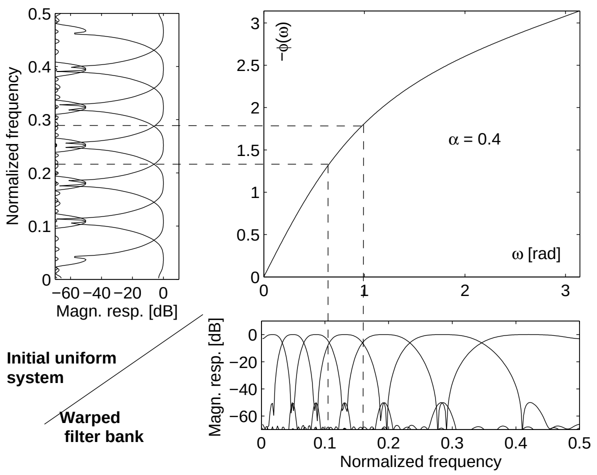


Fig. 3 Idea of allpass transformation

This is done by allpass transformation, i.e. replacing each delay in initial uniform system with stable and casual allpass filter

$$z^{-1} \quad \Rightarrow \quad A(z) = \frac{z^{-1} - \alpha}{1 - \alpha z^{-1}} \qquad |\alpha| < 1 \qquad (5)$$

having nonlinear phase response

$$A\left(e^{j\omega}\right) = e^{j\phi(\omega)} \quad \phi(\omega) = -\omega + 2\arctan\frac{\alpha\sin\omega}{\alpha\cos\omega - 1} \quad (6)$$

of the behavior depending on the value of coefficient α.

From the mathematical point of view the bilinear conformal mapping of the unit circle to itself is done that way. The concept is similar to the well-known frequency transformations for digital filters.

With a suitable selection of the coefficients of the transformation and higher order allpass filters an arbitrary frequency partitioning is obtainable [4]. Among others, good approximation of psychoacoustic scales can be achieved [5]. The recently proposed warped variant of pseudo-QMF system [6] seems to be very promising for audio applications. Nevertheless, allpass transformed DFT filter banks have been successfully used in speech enhancement systems [7, 8].

Nonsubsampled warped filter banks perform well – all distortions can be minimized by appropriate prototype design and phase equalization. But the decimation of highly redundant channel signals is desired. However there is no question of integer band positioning and hence critical subsampling. Only oversampling at certain levels is permitted. Otherwise, spectral aliasing causes bumps or dips in amplitude response. Fig. 4 explains this phenomenon for cosine modulated filter bank.

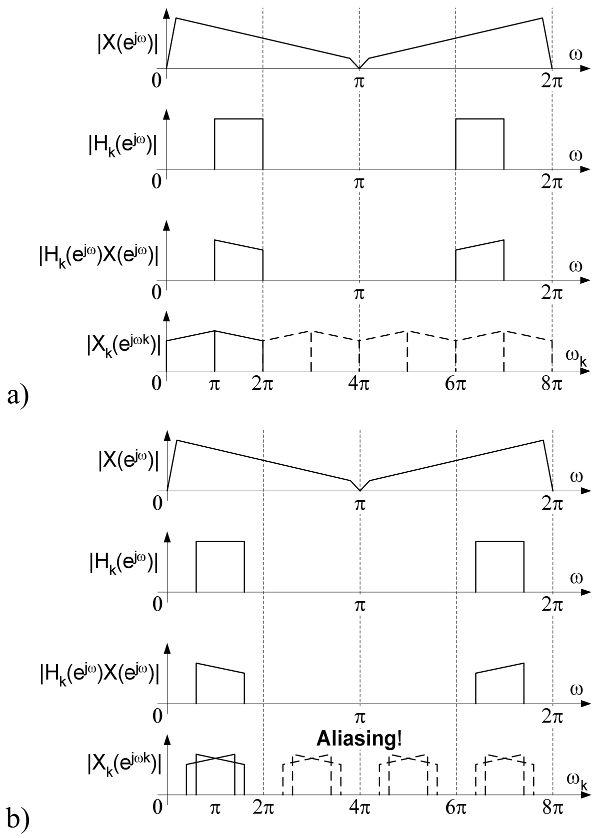


Fig. 4 Subsampling of integer (a) and arbitrary (b) positioned subband in cosine modulated filter bank

As the channel signal is real-valued its spectrum consists of two symmetric parts. If the subband is integer positioned, then critical sampling with the ratio (equal to 4 in the picture) determined by the bandwidth is permitted as there is no overlapping, what is shown in Fig. 4a. After the displacement of the subband, subsampling with the same rate becomes out of place. The replicas of one significant spectral component partially cover those of the latter causing distortions as in Fig. 4b.

3 Rule of selection of subsampling ratios

Lets assume we have the bank of K non-overlapping ideal real filters of different bandwidths covering the entire frequency range. The non-ideality of the filters is neglected for the sake of clarity as we are concentrated on the subsampling process. We also restrict our considerations to the simplest subsampling scheme in the sense of the decimation of the outputs of the analysis filter bank and the expansion of the inputs of the synthesis one as in Fig. 2. Other approaches are disallowed as making the system structurally inconsistent.

The task is to find the integer values of the subsampling ratio $S_k, k = 0..K-1$ for each channel of such a filter bank, guarantying exact reconstruction when assuming ideal filters.

After filtering, each of the subbands maintains the same frequency f_s as the input signal. The ratio of subsampling is equivalent for the quotient

$$S_k = f_s / f_{sk} \tag{7}$$

where f_{sk} is new, lower than f_s, sampling rate of the kth channel. So knowing from (2) the lowest f_{sk}, we can determine the upper bound for sampling ratio as

$$S_k \le \left\lfloor \frac{f_s}{\min(f_{sk})} \right\rfloor = \left\lfloor \frac{f_s n_k}{2 f_{Uk}} \right\rfloor \qquad n_k = \left\lfloor \frac{f_{Uk}}{B_k} \right\rfloor \tag{8}$$

This value potentially offers the maximal redundancy elimination as it is related to the minimal oversampling. However, it is not always valid as other constraints must be satisfied.

The exact formula for the allowed ratios can be constructed by rewriting (4) for f_{sk} and substituting (7) to get

$$\frac{n_k}{2 f_{Uk}} f_s \ge S_k \ge \frac{n_k - 1}{2 f_{Lk}} f_s \tag{9}$$

These inequalities are expressed graphically as the shaded areas between the hyperboles in Fig. 5 equivalent to Fig. 1 but allowing the direct determination of subsampling ratio instead of sampling frequency.

For example, we can apply this plot to the 16th subband of filter bank considered later in Sec. 4. The bandwidth is 0.0612 and the upper frequency is 0.4345 (both normalized by f_s). To determine S_{16}, we analyze the intersections of $n = 7$ horizontal lines at the levels of the multiplies of the bandwidth, with the vertical line placed in accordance with the subband location. Only three intersections (marked with triangles) fall into the valid regions thus the corresponding subsampling ratios are safe. The ratios related to the intersections (marked with circles) lying between the shaded zones are invalid.

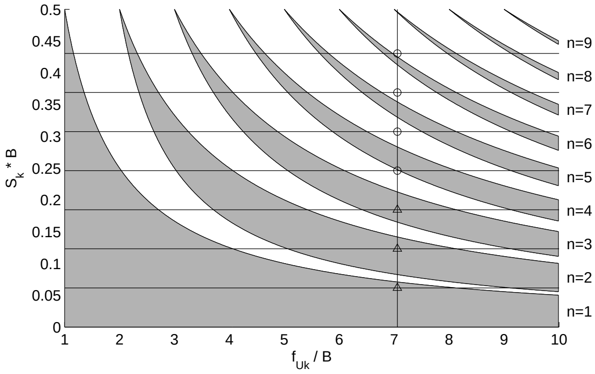


Fig. 5 Allowed subsampling ratios vs. subband position

This is explained in Fig. 6. It shows the integer frequency partitionings (short vertical lines) corresponding to consecutive values of subsampling ratio (one row of lines). Additionally, the range of the 16th subband is put on the plot as horizontal lines.

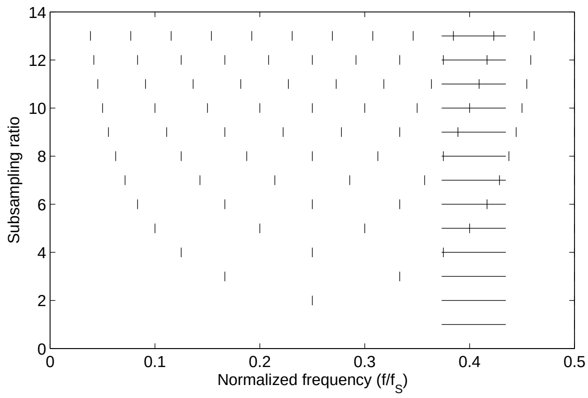


Fig. 6 Relation between 16-th subband and integer band positions corresponding to subsampling

For the acceptable values of S_{16} the subband is located inside one of the integer positions determined by the subsampling ratio. Thus as in the situation in Fig. 4a spectra overlapping doesn't occur.

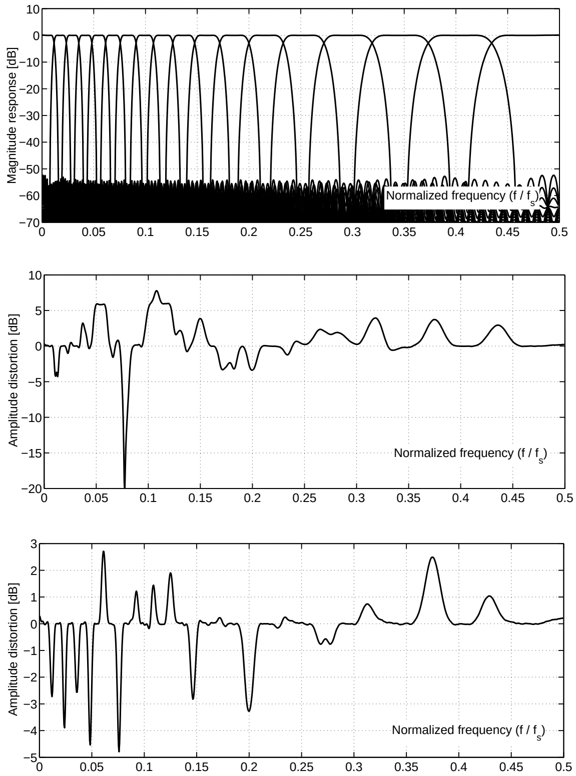
In the case of unsafe (but satisfying (8)) subsampling ratios, the subband occupies fragments of two integer positions. This is graphically represented by the intersections of the lines. The consequence of that is aliasing shown in Fig. 4b.

Finally, for the subsampling ratios violating (8), the bandwidth is wider than the interval of integer partitioning so the intersections are unavoidable.

4 Experimental results

Lets consider the example of warped filter bank approximating the psychoacoustic Bark scale for the speech enhancement module for telephony or voice control [8]. Assuming the standard sampling frequency of 8 kHz, there are 18 channels and the suitable value of the warping coefficient is 0.4092 (according to [5]). The order of the prototype FIR filter is 216.

The channel frequency responses are presented in Fig. 7a.



a) b) c)

Fig. 7 Characteristics of designed filter bank

In turn the detailed information about the subbands together with the maximal safe subsampling ratios determined in accordance with (9) are arranged to form Table 1. For comparison we show the subsampling ratios determined in accordance with the most conventional lowpass sampling theorem assuming that each subband is translated to the origin of the frequency axis

$$S_k^{\mathrm{LP}} = \lfloor f_{\mathrm{s}}/2B_k \rfloor \qquad (10)$$

The response of analysis-synthesis system for the ratios (10) independent of band positions is shown in Fig. 7b. It suffers from irregular distortions due to aliasing. With safe subsampling the ripples are smaller in amplitude and they appear only at the transition bands as can be seen in Fig. 7c. Their presence is not perceptually audible. They can be reduced using better prototype and extending the rule (9) to respect prototype imperfections (mainly the existence of transition band).

Table 1

Details of subbands and their subsampling

k	$f_{\mathrm{U}k}/f_{\mathrm{s}}$	B_k/f_{s}	$\max(S_k)$	S_k^{LP}
0	0.0116701	0.0116701	42	42
1	0.0234881	0.0118180	21	42
2	0.0356085	0.0121204	28	41
3	0.0482000	0.0125915	31	39
4	0.0614536	0.0132536	32	37
5	0.0755934	0.0141398	33	35
6	0.0908892	0.0152958	27	32
7	0.1076738	0.0167846	23	29
8	0.1263646	0.0186908	19	26
9	0.1474911	0.0211265	20	23
10	0.1717267	0.0242356	17	20
11	0.1999190	0.0281922	15	17
12	0.2330993	0.0331804	8	15
13	0.2724273	0.0393279	11	12
14	0.3189801	0.0465528	6	10
15	0.3732736	0.0542935	8	9
16	0.4345144	0.0612408	3	8
17	0.5000000	0.0654856	7	7

Summing the reciprocals of the ratios we obtain the total oversampling ratio of the system. For critical sampling it would be unity. For the ratios (10) it amounts to 1.0387 and 1.4934 for safe ones. The latter value evidently differs from unity but it looks very good in comparison to the case of nonsubsampled system with data multiplied 18 times.

5 Conclusions

An appropriate selection of subsampling ratios is of importance for the exact reconstruction property of non-uniform filter bank and the efficiency of in-band processing. The presented rule being the simple adoption of the bandpass sampling theorem explicitly gives the set of suitable ratios. A system designed in accordance with the given principles well reconstructs passed signal though channels are only slightly oversampled (redundancy is moderate). The further reduction of the distortions requires deeper consideration of the prototype's properties what can be the subject of further research.

References

[1] R.G. Vaughan, N.L. Scott, D.R. White, "The Theory of Bandpass Sampling", IEEE Trans. Signal Processing, Vol. 39, No. 9, Sept. 1991, pp. 1973-1984

[2] C. Herley, P.W. Wong, "Minimum Rate Sampling and Reconstruction of Signals with Arbitrary Frequency Support", IEEE Trans. Inform. Theory, Vol. 45, No. 5, July 1999, pp. 1555-1564

[3] P. P. Vaidyanathan, "Multirate Systems and Filter Banks", PTR Prentice Hall, Englewood Cliffs, New Jersey, 1993
[4] M. Kappellan, B. Strauss, P. Vary, "Flexible nonuniform filterbanks using allpass transformation of multiple order", Proc. EUSIPCO'96, Trieste, Italy, Sep. 1996, pp. 1745 – 1748
[5] J.O. Smith, J.S. Abel, "Bark and ERB Bilinear Transforms", IEEE Trans. Speech, Audio Processing, Vol. 7, No. 6, 1999, pp. 697-708
[6] A. Petrovsky, M. Parfieniuk, K. Bielawski, "Psychoacoustically Motivated Nonuniform Cosine Modulated Polyphase Filter Bank", Proc. SMMSP'2002, Toulouse, France, Sept. 2002, pp. 95 – 101
[7] T. Gulzow , A. Engelsberg, U. Heute, "Comparison of a discrete wavelet transformation and nonuniform polyphase filterbank applied to spectral-subtraction speech enhancement", Signal Processing, Vol. 64, 1998, pp. 5-19
[8] K. Bielawski, A.A. Petrovsky: "Speech enhancement system for hands-free telephone based on the psychoacoustically motivated filter bank with allpass frequency transformation", Proc. EUROSPEECH'99, Budapest, Hungary, Sept. 1999, pp. 2555-2558

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