Reduced Complexity Synthesis Part of Non-Uniform Near-Perfect-Reconstruction DFT Filter Bank Based on All-Pass Transformation*

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Abstract - **This paper presents some improvements to the known structure of the synthesis part of near-PR non-uniform all-pass transformed DFT filter bank. The resulting systems are functionally equivalent to the base schema but their complexity is considerably reduced in terms of computations and storage. Moreover, their mathematical descriptions lead to alternative, simpler or more convenient forms of conditions of exact reconstruction. Finally, more efficient (but less flexible) construction of phase compensation filter is proposed.**

1 INTRODUCTION

In recent years, warped filter banks based on all-pass frequency transformation [1] are considered as alternative tool for non-uniform spectral decomposition. Their most important advantages are unusual flexibility and ease of unequal bandwidth shaping. Among other things, they allow simple and good approximation of critical bands of hearing [2]. This property was successfully exploited in speech enhancement systems basing on perceptual rules [3]. But on the other side, the nonuniformity of these systems causes serious difficulties in obtaining perfect reconstruction (PR). The realization of the synthesis bank by direct inversion of the analysis stage is impractical as it leads to instability problems. The known stable solutions, in turn, suffer from phase distortions.

This problem was addressed recently in the works [4, 5], in which near-perfect reconstruction warped filter bank was developed. On the one side, a novel method of phase equalization was shown, based on stable FIR subband filters at synthesis side. Also lifting schemes were incorporated into warped system to enlarge the number of coefficients and to increase frequency selectivity without disturbing the reconstruction quality. When we tried to apply the first concept to our solution of warped pseudo-QMF filter bank [6], we noticed the perspectives of some simple but significant improvements leading to reduced complexity, which are presented below.

2 ALL-PASS TRANSFORMED ANALYSIS FILTER BANK

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The all-pass transform [1] is a simple method of obtaining non-uniform filter bank. It consists in replacing all the delays in uniform polyphase DFT modulated system with identical, causal and stable, all-pass filters

$$z^{-1} \quad \Rightarrow \quad A(z) = \frac{z^{-1} - a}{1 - az^{-1}} \qquad |a| < 1 \tag{1}$$

Such a filter has unity magnitude response and nonlinear phase

$$A\left(e^{j\omega}\right) = e^{j\phi(\omega)} \quad \phi(\omega) = -\omega + 2\arctan\frac{a\sin\omega}{a\cos\omega - 1} \tag{2}$$

As the all-pass transfer function represents conformal bilinear mapping of the unit circle to itself, the frequency scale of resulting filter bank is deformed, so its frequency response becomes warped. The character of the deformation depends only on the value of the coefficient a and is explicitly given by the curve depicting negative phase $-\phi(\omega)$. This is explained in Fig. 1.

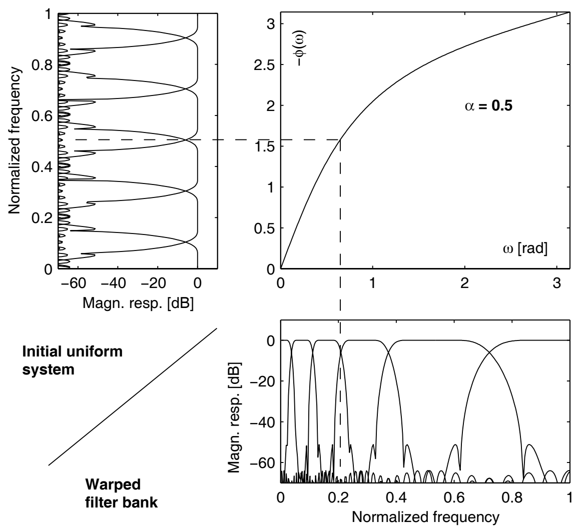

Figure 1: Filter bank frequency response warping with all-pass transform.

The structure of all-pass transformed analysis DFT filter bank is shown in Fig. 2. For the case of K-channel system based on the prototype $P(z)$, the warping of the frequency responses of the subband filters $H_k(z), k = 0...K-1$ can be formulated as

$$H_k\left(e^{j\omega}\right) = P\left(e^{j\left(\omega - \frac{2\pi}{K}k\right)}\right) \Rightarrow H_k\left(e^{-j\phi(\omega)}\right) = P\left(e^{-j\left(\phi(\omega) + \frac{2\pi}{K}k\right)}\right) \tag{3}$$

Obviously, the channel signals can be decimated to reduce their redundancy. But in the following, for the sake of brevity, we assume that there is no subsampling.

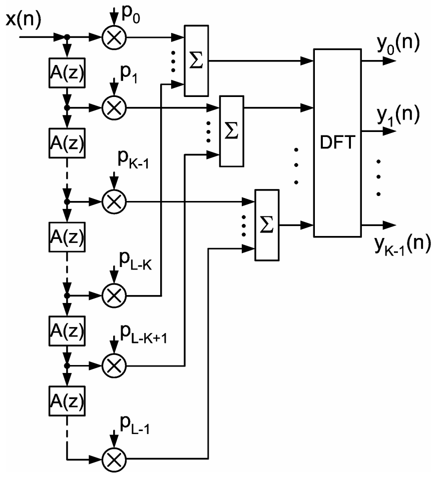

Figure 2: Base analysis structure.

3 MATRIX POLYPHASE REPRESENTATION OF TRANSFER FUNCTION OF ANALYSIS-SYNTHESIS SYSTEM

The consideration of different structures of synthesis filter bank must be introduced with defining of some additional notations and assumptions.

For clarity we confine ourselves to K-channel filter banks based on linear phase FIR prototype filter of order $L = mK$. As different prototypes may be used in analysis and synthesis, their coefficients are designated as $p(n)$ and $q(n)$, $n = 0..L-1$, accordingly.

The transfer function of overall analysis-synthesis system can be represented by the following product

$$T(z) = \underbrace{T_{PF}(z)}_{\text{post-filter}}\underbrace{\mathbf{r}(z)\mathbf{R}(z)\mathbf{W}}_{\text{synthesis stage}}\underbrace{\mathbf{W}^H\mathbf{E}(z)\mathbf{e}(z)}_{\text{analysis stage}} \qquad (4)$$

This is a common framework for all following considerations. The factors related to the analysis part of the structure depicted in Fig. 2 are

$$\mathbf{e}(z) = \left[1, A(z), \ldots, A^{K/2-1}\right]^T \qquad (5a)$$

$$\mathbf{E}(z) = \begin{bmatrix} \operatorname{diag}\left[E_0\left(A^K(z)\right), \ldots, E_{K/2-1}\left(A^K(z)\right)\right] \\ A^{K/2}(z)\operatorname{diag}\left[E_{K/2}\left(A^K(z)\right), \ldots, E_{K-1}\left(A^K(z)\right)\right] \end{bmatrix} \qquad (5b)$$

where

$$E_\rho\left(A^K(z)\right) = \sum_{\lambda=0}^{m-1} p(\lambda K + \rho) A^{\lambda K}(z) \qquad (5c)$$

are all-pass transformed polyphase components of the prototype. The matrices $\mathbf{W}$ and $\mathbf{W}^H$ (H denotes Hermitian transposition) describe DFT and its inverse – as their product is an identity matrix, it can be omitted in all further calculations.

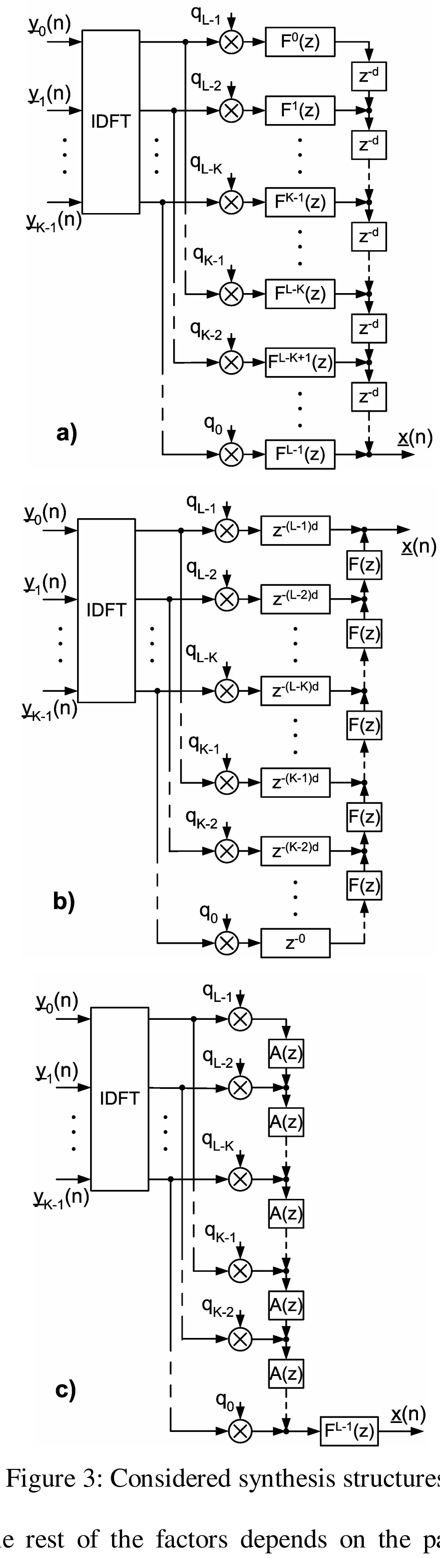

Figure 3: Considered synthesis structures.

The rest of the factors depends on the particular structure of the synthesis part. Nevertheless, in all cases, $\mathbf{R}(z)$ is a horizontal concatenation of two diagonal matrices

$$\mathbf{R}(z) = \left[\mathbf{R}_L(z) \quad \mathbf{R}_R(z)\right] \qquad (6)$$

In turn $T_{PF}(z)$ represents an additional post-filtering of total output.

4 KNOWN SOLUTION FOR NEAR-PR SYNTHESIS FILTER BANK

Our goal is to approximate PR, i.e. to achieve total response close to pure delay

$$T(z) \approx z^{-D} \tag{7}$$

by appropriate construction of the synthesis stage. The trivial simple inversion is not a valid solution as $A(z)^{-1}$ is unstable. However, the special synthesis scheme proposed in [4] and depicted in Fig. 3a, allows arbitrary good approximation of PR, at the cost of increased system delay. It is based on the observation that the function of the form

$$F(z) = \left(1 - az^{-1}\right) \cdot \left(z^{-(d-1)} + z^{-(d-2)}a + ... + z^{-1}a^{d-2} + a^{d-1}\right) \tag{8}$$

can compensate the phase distortions introduced by all-pass. The product

$$\begin{gathered} A(z)F(z) = \frac{z^{-1} - a}{1 - az^{-1}} \cdot \left(1 - az^{-1}\right) \cdot \\ \cdot \left(z^{-(d-1)} + z^{-(d-2)}a + ... + z^{-1}a^{d-2} + a^{d-1}\right) = z^{-d} - a^d \end{gathered} \tag{9}$$

converges to pure delay with the increase in d, as the second term (representing compensation error) decay to zero for $|a| < 1$. The factor d guarantying desired level of distortion cancellation evidently depends on all-pass transform coefficient. For $|a|$ near 0, the term a^d is negligible already for small values of the exponent. Otherwise, when $|a|$ is close to 1, the equivalent reduction in the error requires a much longer delay.

For this approach, the missing components of the overall transfer function (4) are

$$\mathbf{r}(z) = \left[z^{-\left(\frac{K}{2}-1\right)d}, z^{-\left(\frac{K}{2}-2\right)d}, ..., 1\right] \tag{10a}$$

$$\mathbf{R}_L(z) = z^{-\frac{K}{2}d} \operatorname{diag}\left[R_{K-1}(z), ..., R_{\frac{K}{2}}(z)\right] \tag{10b}$$

$$\mathbf{R}_R(z) = \operatorname{diag}\left[R_{\frac{K}{2}-1}(z), ..., R_0(z)\right] \tag{10c}$$

where

$$R_\rho(z) = \sum_{\lambda=0}^{m-1} q(\lambda K + \rho) F^{K(m-\lambda)-\rho-1}(z) z^{-K\lambda d} \tag{10d}$$

As there is no post-filtering, $T_{PF}(z) = 1$. The corresponding near PR conditions are

$$\mathbf{R}(z)\mathbf{E}(z) = \frac{2}{K} z^{-s} \operatorname{diag}\left[1, F^1(z), ..., F^{\frac{K}{2}-1}(z)\right] \tag{11}$$

where the s parameter is determined by the assumed overall system delay D [4].

The approach is good and allows designing warped filter banks with negligible distortions. However, the high computational cost of the above synthesis structure seems to obstruct practical applications. It is evident that the function $F(z)$ is computed $\frac{L}{2}(L-1)$ times for each processed sample. Moreover, the order of $F(z)$ depends directly on the factor d, so it may be high for good compensation of the phase distortion. Thus, for large d and long prototype impulse response, the computational cost of the considered synthesis structure is enormous.

5 ALTERNATIVE STRUCTURE OF REDUCED COMPUTATIONAL LOAD

It may be noticed that the delays can be interchanged with the phase compensators. This leads to the structure in Fig. 3b. Now the items describing the synthesis filter bank are

$$\mathbf{r}(z) = \left[F^{\frac{K}{2}-1}(z), ..., F^1(z), 1\right] \tag{12a}$$

$$\mathbf{R}_L(z) = F^{\frac{K}{2}} \operatorname{diag}\left[R_0(z), ..., R_{\frac{K}{2}-1}(z)\right] \tag{12b}$$

$$\mathbf{R}_R(z) = \operatorname{diag}\left[R_{\frac{K}{2}}(z), ..., R_{K-1}(z)\right] \tag{12c}$$

with

$$R_\rho(z) = \sum_{\lambda=0}^{m-1} q(\lambda K + \rho) F^{K(m-1-\lambda)}(z) z^{-(\lambda K + \rho)d} \tag{12d}$$

There is no post-filtering again, so $T_{PF}(z) = 1$. And here the conditions of exact reconstruction can be stated as

$$\mathbf{R}(z)\mathbf{E}(z) = \frac{2}{K} z^{-s}\mathbf{I} \tag{13}$$

where $\mathbf{I}$ is identity matrix of size $K/2$.

Owing to the cascading of the compensation filters to form a chain, $F(z)$ is evaluated only $L-1$ times. However, the number of the delays in subbands is still large - $\frac{L}{2}(L-1)d$ total, so memory requirements remain high.

6 ALTERNATIVE STRUCTURE WITH POST-FILTERING

The final approach is to extract phase compensation filters to the system output as Fig. 3c shows. Thus the post-filter is formed

$$T_{PF}(z) = F^{L-1}(z) \tag{14}$$

and all delay sequences (occupying memory) disappear from the synthesis structure, which become symmetric to analysis stage.

$$\mathbf{r}(z) = \left[A^{\frac{K}{2}-1}(z), ..., A(z), 1\right] \tag{15a}$$

$$\mathbf{R}_L(z) = A^{\frac{K}{2}}(z) \operatorname{diag}\left[R_{K-1}(z), ..., R_{\frac{K}{2}}(z)\right] \tag{15b}$$

$$\mathbf{R}_R(z) = \operatorname{diag}\left[R_{\frac{K}{2}-1}(z), ..., R_0(z)\right] \tag{15c}$$

$$R_\rho\left(A^K(z)\right) = \sum_{\lambda=0}^{m-1} q(\lambda K + \rho) A^{\lambda K}(z) \tag{15d}$$

It is evident that polyphase components are much simpler. Moreover, the condition of exact reconstruction

$$\mathbf{R}(z)\mathbf{E}(z) = \frac{2}{K} A^{L-K/2}(z)\mathbf{I} \tag{16}$$

clarifies, as neither $F(z)$ nor d is involved.

The consequence of this approach is in moderate increase in computational complexity as all-pass chain must be computed at the synthesis side.

In Table 1 all three considered synthesis structures are compared in terms of computational complexity and memory requirements of filtering part (without IDFT). It is evident that the post-filtering represents a reasonable compromise between the requirements of both resources.

Struct.	Real multiplications	Memory cells
Fig. 3a	$2L+L(L-1)d$	$L(L-1)d+2(L-1)d$
Fig. 3b	$2L+2(L-1)d$	$L(L-1)d+2(L-1)d$
Fig. 3c	$2L+4(L-1)+2(L-1)d$	$2(L-1)+2(L-1)d$

Table 1: Comparison of complexity.

7 MORE EFFICIENT PHASE COMPENSATION FILTER

Another step deals with the function $F(z)$ alone. If we restrict d to be power of two, then the following elementary factorization holds

$$F(z)=\left(1-az^{-1}\right)\cdot\left(z^{-1}+a\right)\cdot\left(z^{-2}+a^{2}\right)\cdot\ldots \\ \ldots\cdot\left(z^{-d/4}+a^{d/4}\right)\cdot\left(z^{-d/2}+a^{d/2}\right) \qquad (17)$$

which still gives

$$A(z)F(z)=\frac{z^{-1}-a}{1-az^{-1}}\cdot\left(1-az^{-1}\right)\cdot \\ \cdot\left(z^{-1}+a\right)\cdot\left(z^{-2}+a^{2}\right)\cdot\ldots\cdot\left(z^{-d/4}+a^{d/4}\right)\cdot \\ \cdot\left(z^{-d/2}+a^{d/2}\right)=z^{-d}-a^{d} \qquad (18)$$

Hence, instead of implementing the full polynomial $\left(z^{-(d-1)}+z^{-(d-2)}a+\ldots+z^{-1}a^{d-2}+a^{d-1}\right)$, it is possible to implement the cascade of $\log_2 d$ binomials. As each binomial requires only one multiplication, the total number of evaluations decreases.

8 PRACTICAL EXAMPLE

As an example of all-pass phase nonlinearity compensation based on the considered approach, the results obtained for the structure with post-filtering, described in Sec. 6, are depicted in Fig. 4. The system contained the chain of 24 all-passes with the coefficient $a=0.3$. The plots show that $d\approx 8$ guarantees complete equalization of phase response and group delay.

9 CONCLUSIONS AND FUTURE WORKS

The presented improvements are valuable when the practical implementation of warped filter bank is considered, especially for real time applications. They are suited for warped filter banks with as DFT as cosine modulation. But the latter case requires more formalization and now is still under development.

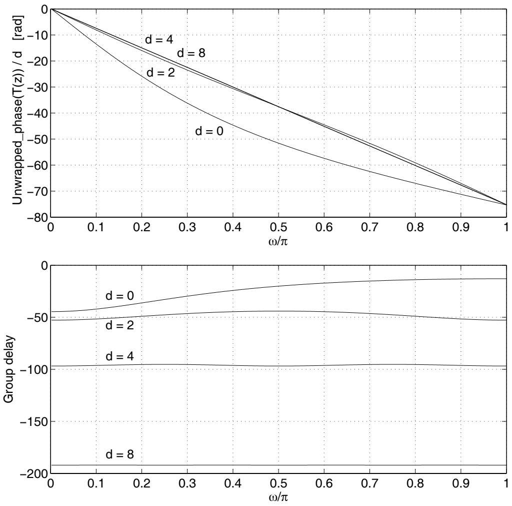

Figure 4: Example of phase equalization.

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